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Analysis of State Estimation Technique for a Quadruple Tank Process.

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ABSTRACT

Quadruple tank (QT) is a benchmark problem for a multivariable nonlinear interactive tank system. Estimation is the process of approximating the state of a system, to estimate the levels of the QT minimizing the effect of noise emanating from the sensor that degrades the controller performance. The Conventional state estimators used to estimate the true state of QT system from noisy sensor information has limitations such as model approximation and computational complexity. Kalman Filter (KF) widely known optimal estimator, it is limited only to linear processes. Extended Kalman Filter (EKF) is an extended form of KF used for nonlinear process. Unscented Kalman Filter (UKF) a modified form of EKF, addresses the shortcomings of Extended Kalman Filter (EKF). This investigation presents the comparative analysis of EKF and UKF based state estimation for QT that overcomes the disadvantages of the conventional estimators. Simulation of EKF and UKF based state estimation of QT is carried out in MATLAB software and the result indicates that the UKF provides the best estimation.

Keywords: Quadruple tank, State Estimation, Kalman Filter, Extended Kalman Filter, Unscented Kalman Filter

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INTRODUCTION

A Quadruple tank is a nonlinear system with Multi input Multi output (MIMO). The basic mathematical modelling of QT in [1] gives the dynamics of the system behaviour. To estimate the level in QT sensor produces some internal noise problem is less possible to eliminate it from the system performance, so there is a necessity for estimating the system performance. The noise produced in sensor, will affect data accuracy in the process and hence there exist a poor controller performance thus the state estimation is necessary for the process to improve the overall performance by providing a better controller.

State estimation of QT is discussed in various research works that Kalman filter is a linear process that faces the difficulties in solving the nonlinear function [2]. The two interconnected tanks are estimated using the EKF algorithm in [3] gives satisfactory result. For dynamic signals used in the communication system is analysed in [4] are widely corrupted with the noise, if the SNR ratio is low and suggested that PSO produces the better result than EKF.

Unscented Kalman Filter gives an effective solution for the system deals with approximating the complex state [5]. A new method has been implemented in [6] known as Unscented filtering (UF) will provide an easier way to find approximate the Gaussian curve. The UF allows the mean and covariance to propagate through the nonlinear function and find out the new mean and covariance at the output side. Unscented Transformation (UT), that allows to select the sigma points has to be around the mean of the input Gaussian curve [7] these point are propagated through the nonlinear function and gives out an new form of mean and covariance. Unscented Transformation (UT) technique is used for proceeding the UKF algorithm. For maintain the level of QT estimation plays the vital role, also in order to improve the performance of the system the response are been estimated using these filtering algorithm.

The rest of the paper is structured as, Description of the Quadruple tank process in section II, The basic mathematical modelling is explained in section III, Dynamic behaviour of the system in section IV, Unscented Kalman filter algorithm is discussed in the section V, Simulated result in section VI, finally Conclusion in section VII.

QUADRUPLE TANK DESCRIPTION

A Quadruple Tank system consist of four interconnected tanks (T_1, T_2, T_3 and T_4) out of that two tanks (T_1 and T_2) are placed at the bottom of (T_3 and T_4) respectively, a reservoir is kept to store the liquid to feed the tanks. Two pumps (P_1 and P_2) are been provided to feed the tanks for the applied voltages V_1 and V_2 , with the flow ratio of γ_1 and γ_2 . Pump P_1 feeds the tanks T_1 and T_4 respectively for given flow ratio γ_1 , similarly pump P_2 feeds the tank T_2 and T_3 respectively for given flow ratio γ_2 . The parameters of the tanks are noted as follows the area of cylindrical tanks are (A_1, A_2, A_3 , and A_4), the outlet of each tanks is represented as (a_1, a_2, a_3 and a_4), the level of each tank is denoted as (h_1, h_2, h_3 and h_4).

Due to various flow rate applied to the QT system there exist an interaction problem and the heights in each tanks get varied. QT has two different types of operating regions, they are Minimum Phase Mode (MPM) and the Non Minimum Phase Mode (NMPM).

MPM will allow the flow of liquid in the upper tank is higher than the lower tank, in order to archive MPM the flow ratio sum must be more than 0 but less than 1, NMPM will allow the flow of liquid in the lower tank is higher than the upper tank, to implement NMPM the flow ratio of QT sum must be more than 1 but less than 2.

Table 1: Flow rate to the tanks generated by the pump

Pump	Tank 1	Tank 2	Tank 3	Tank 4
1	$\gamma_1 k_p v_1$	-	-	$(1 - \gamma_1) k_p v_1$
2	-	$\gamma_2 k_p v_2$	$(1 - \gamma_2) k_p v_2$	-

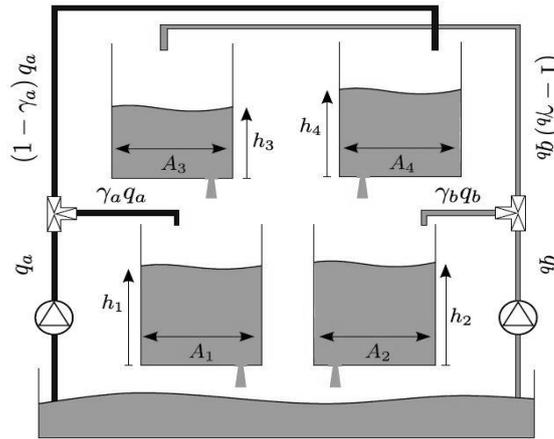


FIG 1: SCHEMATIC REPRESENTATION OF QUADRUPLE TANK SYSTEM.

MATHEMATICAL MODEL OF QUADRUPLE TANK

From the description the physical parameter of the QT system the mathematical model is derived from the Mass Balance equation and Bernoulli’s equation using the inputs and outputs in the following section.

Mass Balance equation

The rate of flow is equal to the difference between the inflow rate and the outflow rate of the tank i.e., Rate of accumulation = (inflow rate – outflow rate)

The mass balance equation is given independently for all the four tanks they are as follows

$$\dot{V} = A * \dot{h} \tag{1}$$

$$\dot{V} = q_{in} - q_{out} \tag{2}$$

$$A_i * \dot{h} = q_{in} - q_{out} \tag{3}$$

Where, \dot{V} is the volume of liquid in cm^3 , A is the cross sectional area of the tank in cm^2 , h is the level of liquid in cm , q_{in} is inflow rate of liquid to the tank in cm^3/s , q_{out} is the outflow rate of the tank in cm^3/s .

3.2 Bernoulli’s equation

The Bernoulli’s equation is evaluated for an incompressible liquid and it is given as

$$p + \frac{1}{2} \rho v_w^2 + \rho gh = constant \tag{4}$$

At the surface of the liquid the velocity $v_w = 0$ and at the bottom the height $h = 0$ Substituting the condition to the Bernoulli’s equation we get

$$p + \rho gh = c \tag{5}$$

$$p + \frac{1}{2} \rho v_w^2 = c \tag{6}$$

On subtracting the equation (5) and (6) we get

$$v_w^2 = 2gh \tag{7}$$

$$v_w = \sqrt{2gh} \tag{8}$$

$$q_{out} = a * v_w \tag{9}$$

$$q_{out} = a * \sqrt{2gh} \tag{10}$$

Where, a is cross sectional area of the outlet in cm^2 , v_w is the speed of liquid at outlet flow rate in lph, h is the level of liquid in cm, ρ is Density of the liquid in Kg/cm^3 , p is pressure of the liquid flow in N/m^2 , g is acceleration due to gravity in cm/s^2 .

The pump generated flow is given by $q_{\text{pump}} = k_p v_i$, where $i=1,2$.

The non linear model of quadruple tank is given below

$$A_i \dot{h} = q_{in} - q_{out} \quad \text{where } i=1,2,3,4 \quad (11)$$

$$q_{in1} = \gamma_1 k_p v_1 \quad (12)$$

$$q_{in2} = \gamma_2 k_p v_2 \quad (13)$$

$$q_{in3} = (1 - \gamma_2) k_p v_2 \quad (14)$$

$$q_{in4} = (1 - \gamma_1) k_p v_1 \quad (15)$$

$$q_{out} = a_i v_w = a_i \sqrt{2gh_i} \quad \text{where } i=1,2,3,4 \quad (16)$$

For tank 1

Rate of accumulation = (inflow rate – outflow rate)

$$A_1 \dot{h}_1 = q_{in1} + q_{out3} - q_{out1} \\ A_1 \dot{h}_1 = \gamma_1 k_p v_1 + a_3 \sqrt{2gh_3} - a_1 \sqrt{2gh_1} \quad (17)$$

For tank 2

Rate of accumulation = (inflow rate – outflow rate)

$$A_2 \dot{h}_2 = q_{in2} + q_{out4} - q_{out2} \\ A_2 \dot{h}_2 = \gamma_2 k_p v_2 + a_4 \sqrt{2gh_4} - a_2 \sqrt{2gh_2} \quad (18)$$

For tank 3

Rate of accumulation = (inflow rate – outflow rate)

$$A_3 \dot{h}_3 = q_{in3} - q_{out3} \\ A_3 \dot{h}_3 = (1 - \gamma_2) k_p v_2 - a_3 \sqrt{2gh_3} \quad (19)$$

For tank 4

Rate of accumulation = (inflow rate – outflow rate)

$$A_4 \dot{h}_4 = q_{in4} - q_{out4} \\ A_4 \dot{h}_4 = (1 - \gamma_1) k_p v_1 - a_4 \sqrt{2gh_4} \quad (20)$$

The non linear differential equation of all four tanks is as follows

$$\dot{h}_1 = \frac{1}{A_1} (a_1 \sqrt{2gh_3} + \gamma_1 k_p v_1 - a_3 \sqrt{2gh_1}) \quad (21)$$

$$\dot{h}_2 = \frac{1}{A_2} (a_2 \sqrt{2gh_4} + \gamma_2 k_p v_2 - a_4 \sqrt{2gh_2}) \quad (22)$$

$$\dot{h}_3 = \frac{1}{A_3} (1 - \gamma_2) k_p v_2 - a_3 \sqrt{2gh_3} \quad (23)$$

$$\dot{h}_4 = \frac{1}{A_4} (1 - \gamma_1) k_p v_1 - a_4 \sqrt{2gh_4} \quad (24)$$

Where, A cross-section area of Tank in cm^2 , a cross-section area of the outlet of Tank in cm^2 , h liquid level of Tank in cm, v pump voltage in V, g acceleration due to gravity cm/s^2 .

By knowing the mathematical model of QT the dynamic of the system can be studied in the next section.

DYNAMICS OF QUADRUPLE TANK

In order to study the dynamic behavior of the quadruple tank system can be studied under two cases

- Keeping flow ratio as constant and varying pump voltages
- Keeping pump voltages as constant and varying flow ratio

This dynamic behavior helps in selecting the operating pump voltage and the flow ratio, to maintain the required height of the water in all the tanks, also individual ranges of the tank.

- Keeping flow ratio as constant and varying pump voltages

For a fixed flow ratio of $\gamma_1 = 0.7, \gamma_2 = 0.6$, then the response is noted by varying the pump voltage provides maximum height in each tank is shown in table 2. When the voltage is greater than 2.5V water level reaches 10 cm when compared to the other set of voltage. If any one of the pump voltage is less than 2.5V the level in all the tanks will be less than 10 cm. Voltage can be ranged from 0 to 4 V, when exceeding this range the system tends to unstable. Tanks 3 and 4 get settled much faster than the tanks 1 and 2 this is because these two tanks get two input, one from the pump 1 and the other from the drain of tank 3 so it takes some time interval to settle. Whereas tanks 3 and 4 get only one input that is from the pumps. Maximum level of individual tanks is shown in table 3.

Table 2: Level of all four tanks for an fixed $\gamma_1 = 0.7, \gamma_2 = 0.6$

Tank	V1=3 V2=3	V1=2.5 V2=3	V1=1 V2=3	V1=3.5 V2=3.5	V1=1 V2=1
1	12.2	9.7	4.0	16.6	1.3
2	12.6	11.3	7.6	17.2	1.4
3	0.7	1.1	1.6	11.2	0.17
4	1.4	0.9	0.1	13.6	0.15

Table 3 : Maximum level of individual tanks

$\gamma_1 = 0.7 \ \gamma_2 = 0.6$ (are kept constant)	
Voltage get varies	
TANK	MAX HEIGHT
1	12.2
2	12.68
3	11.2
4	13.6

Keeping pump voltages as constant and varying flow ratio

For a fixed pump voltage $V_1 = 3, V_2 = 3$ then the response is noted by varying the flow ratio provides the maximum height for each tank is shown in table 4. If the γ ranges from 0.6 to 0.8 then the water level is maintained between 10 cm to 22.5 cm. when any one of the γ value is less than 0.5 then the quadruple tank system faces the unstable system situation. When one of the γ value is 0.8 then the other is less than 0.2, tank2 overflow which indicates that the system is set to be unstable. The water level exits the normal level of the tank the values are shown in the table 3. The settling time for the upper tanks is faster than the lower tanks is because it has only one input and one output whereas the lower tanks has two input and one output. A maximum level of the individual tanks for the fixed pump voltage and variable flow ratio is shown in table 5.

Table 4: Level of all four tanks for an fixed $V_1=3, V_2=3$, and variable γ values

Tank	G1=0.2 G2=0.8	G1=0.5 G2=0.5	G1=0.7 G2=0.2	G1=0.9 G2=0.1	G1=0.1 G2=0.1
1	1.6	10	22.7	32.6	12.2
2	39.9	15.6	3.9	0.62	12.6
3	0.40	2.5	6.4	8.1	8.1
4	10.0	3.9	1.4	0.1	10

* G1 and G2 represents the gamma

Table 5: Maximum level of individual tanks

V1=3,V2=3 (are kept constant)	
γ get varies	
TANK	MAX HEIGHT
1	32.69
2	39.99
3	8.17
4	10.01

This study helps in selecting the range of the water level in the tank and to maintain the quadruple tank system as a stable system for a specified set of fixed and variable values.

ALGORITHM

This section discuss about the EKF and the UKF algorithms corresponding steps that involved in processing the filtering techniques. The flow of prediction, correction and updation in both the algorithms has mathematically derived and the sets of equation is given as follows

5.1 Extended Kalman filter algorithm

Extended Kalman filter is the extension version of Kalman filter, in KF it is used only for processing the linear system, when it come in to an non linear system EKF is used. EKF will linearize the non linear function but approximating the function with the help of the Jacobian matrix calculation. The Jacobian Matrix will partially differentiate the equation of the non linear function at each state and then it updates the corresponding state matrix, this process is repeated such that the linearization is archived. The prediction, correction and updation steps are as followed by these equation.

State space equation of the non linear fuction

$$x_k = f(x_{k-1}, u_{k-1}, w_{k-1}) \tag{25}$$

$$y_k = h(x_k, u_k, v_k) \tag{26}$$

Where, f and h are the input, output functions, x_k is the current state of the system , x_{k-1} is previous state, u_{k-1} is control signal given to the previous state, y_k output vector, w_k, v_k is the process and measurement noise.

Calculation of Jacobian Matrix

$$F_{k-1} = \frac{\partial f}{\partial x_{k-1}} , \quad H_{k-1} = \frac{\partial h}{\partial x_k} \tag{27}$$

$$L_{k-1} = \frac{\partial f}{\partial w_{k-1}} , \quad M_k = \frac{\partial f}{\partial v_k} \tag{28}$$

- F, H Differentiated with respect to the state estimate
- L, M Differentiated with respect to the process and measurement noise

Prediction of new state

In the prediction stage the new state of the system is predicted, by solving the non linear equation using the Jacobian matrix. The new Mean and Covariance will form a new state for the system and the previous value will be updated by correcting it with the gain obtained.

$$\hat{x}_{k|k-1} = f(\hat{x}_{k-1}, u_{k-1}, 0) \tag{29}$$

$$P_{k|k-1} = F_{k-1}P_{k-1}F_{k-1}^T + L_{k-1}Q_{k-1}L_{k-1}^T \tag{30}$$

Where, $\hat{x}_{k|k-1}$ is the new expectation of the Predicted state, $P_{k|k-1}$ is the new covariance of the predicted state, P_{k-1} and Q_{k-1} are Process noise covariance matrix and Measurement noise covariance matrix respectively.

Correction in the system

In the correction step the Kalman gain is found, that helps in correcting the deviations that occur in the previous state. The gain is similar to that of error detector in the control system.

$$K_k = P_{k|k-1}H_k^T (H_kP_{k|k-1}H_k^T + M_kR_kM_k^T)^{-1} \tag{31}$$

Updation of the state

Updation is required in order to implement the new values that has been obtained in the correction stage. Without updating the values it is not possible to track the system response.

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k (y_k - h(\hat{x}_{k|k-1}, u_k, 0)) \tag{32}$$

$$P_k = (I - K_kH_k)P_{k|k-1} \tag{33}$$

Unscented Kalman filter algorithm

UKF is a extension algorithm of EKF, it is used for solving the nonlinear process. It is preferred when there exist an higher non linearity in the function, it is capable of solve the problem without linearizing the nonlinear function, and overcomes the complexity that occurs in the EKF, one of the biggest disadvantage in UKF is that it more time to compute the result. UKF involves various steps along with the prediction, correction and updation steps.

Step 1: Generating the Sigma points (χ)

To obtain the sigma point certain scaling parameters are to be calculated , they are

$$\lambda = \alpha^2 (L + \kappa) - L \tag{34}$$

$$\eta_0^m = \frac{\lambda}{(L+\lambda)} \tag{35}$$

$$\eta_0^c = \frac{\lambda}{(L+\lambda)} + 1 - \alpha^2 + \beta \tag{36}$$

$$\eta_i^m = \eta_i^c = \frac{1}{[2(L+\lambda)]}, i = 1, \dots, 2L \tag{37}$$

As the α value is increases the Gaussian curve will spread its curve wider in all direction and gives more possibility to place the sigma points.

The generate the sigma point (2L+1) points has to be selected, where the additional point other than the length of the state is mean.

Where, α Primary scaling parameter, β Secondary scaling parameter, κ Always set to 0, L Length of state vector, η_0^m, η_0^c Weight matrix, λ Additional scaling parameter.

$$\chi = [\bar{x} \quad \bar{x} + \sqrt{L + \lambda} \sqrt{P_x} \quad \bar{x} - \sqrt{L + \lambda} \sqrt{P_x}] \quad (38)$$

Where, \bar{x} primary mean, P_x primary covariance.

Step 2: Transformation χ into a non linear function

$$\chi_{k|k-1}^{(i)} = f(\chi_{k-1}^{(i)}, u_{k-1}), \quad i = 0, 1, \dots, 2L \quad (39)$$

Where, u_{k-1} control signal.

The each column of the sigma point is passed though the nonlinear function along with the control signal and a new transformed sigma point is obtained.

Step 3: To calculate the A prior state estimate

Before the generated sigma point is passed into the nonlinear equation, its normally distributed Gaussian curve mean \hat{x} and covariance $P_{k|k-1}$ is calculated.

Mean

$$\hat{x}_{k|k-1}^{(i)} = \sum_{i=0}^{2L} \eta_i^m \chi_{k|k-1}^{(i)} \quad (40)$$

Where, \hat{x} A prior mean, η_i^m Weight mean matrix, $\chi_{k|k-1}^{(i)}$ Sigma point matrix.

Covariance

$$P_{k|k-1} = Q_{k-1} + \sum_{i=0}^{2L} \eta_i^c (\chi_{k|k-1}^{(i)} - \hat{x}_{k|k-1}^{(i)}) (\chi_{k|k-1}^{(i)} - \hat{x}_{k|k-1}^{(i)})^T \quad (41)$$

Where, Q_{k-1} process noise, η_i^c Weight covariance matrix, $P_{k|k-1}$ A prior covariance

Step 4: Calculate Output sigma point

The new sigma point is calculated along with the Gaussian curves updated mean $\hat{y}_{k|k-1}$ and covariance P_k^{yy}

$$\psi_{k|k-1}^{(i)} = h(\chi_{k|k-1}^{(i)}, u_k), \quad i = 0, 1, \dots, 2L \quad (42)$$

Where, $\psi_{k|k-1}^{(i)}$ newly generated output side sigma point, u_k control signal, h output function.

Mean

$$\hat{y}_{k|k-1} = \sum_{i=0}^{2L} \eta_i^m \psi_{k|k-1}^{(i)} \quad (43)$$

Where, \hat{y} output Mean.

Covariance

$P_k^{yy} =$

$$R_k + \sum_{i=0}^{2L} \eta_i^c (\psi_{k|k-1}^{(i)} - \hat{y}_{k|k-1}^{(i)}) (\psi_{k|k-1}^{(i)} - \hat{y}_{k|k-1}^{(i)})^T \quad (44)$$

Where, P_k^{yy} output covariance, R_k measurement noise.

Step 5: Kalman Gain

The Kalman gain is obtained by combination of auto correlation and cross correlation covariance matrix. The noise is not taken in account for the calculation of cross correlation since it get equalizes the noise value.

Cross covariance matrix

$$P_k^{xy} = \sum_{i=0}^{2L} \eta_i^f (\chi_{k|k-1}^{(i)} - \hat{x}_{k|k-1}^{(i)}) (\psi_{k|k-1}^{(i)} - \hat{y}_{k|k-1}^{(i)})^T \quad (45)$$

Where, P_k^{xy} cross correlation.

Gain Calculation

$$K_k = P_k^{xy} (P_k^{yy})^{-1} \quad (46)$$

Where, K_k Kalman Gain .

Step 6: Update state

$$\hat{x}_k = \hat{x}_{k|k-1} + K_k (y_k - \hat{y}_{k|k-1}) \quad (47)$$

$$P_k = P_{k|k-1} - K_k P_k^{yy} K_k^T \quad (48)$$

Where, \hat{x}_k posterior mean, P_k posterior covariance.

I. SIMULATION RESULT

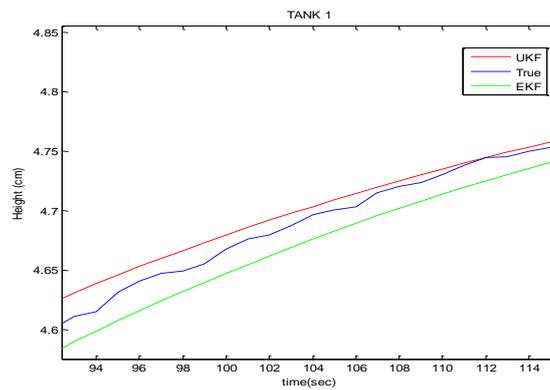


Fig 2: Enlarged view of comparative result of UKF and EKF for Tank 1

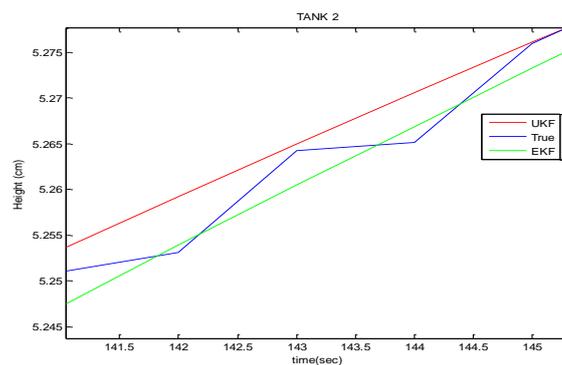


Fig 3: Enlarged view of comparative result of UKF and EKF for Tank 2

For the QT system shown in fig 2 and 3, the parameter values shown in table 6 and 7 are the inputs delivered from the pump to the corresponding tanks tabulated in table 1 is applied with a step input of $V_1=1V$ and $V_2=1V$ along with the flow ratio (γ_1, γ_2) for pump 1 = 0.4 and for pump 2= 0.5. The simulated result shows the level of all the four tanks for the above said input values. The vertical axis give the Height in (Cm) and the horizontal axis shows the Time in (sec). The red line indicates the system response and the blue line indicates the response of UKF along with the sensor noise present in the system.

Table 6: Model parameters of quadruple tank

Parameter	Value	Unit
A_1, A_3	28	cm ²
A_2, A_4	32	cm ²
a_1, a_3	0.071	cm ²
a_2, a_4	0.057	cm ²
K_c	1.00	V/cm
G	981	cm/s ²

Table 7: Scaling parameter of UKF

Parameter	Value
α	10 ⁻⁴ to 1
β	2
ξ	0

CONCLUSION

The Comparative analysis of State estimation of the QT using EKF and UKF algorithms has been implemented. When compared with other conventional estimators, UKF provides better result because it does not linearize the non linear function. Thus approximation is reduced in UKF when compared to EKF algorithm. Higher the approximation in EKF, more the error in the result, thus UKF is better for estimating the non linear equation. In future the noise can be given to the plant and the error can be calculated. Hence the noise is minimized by designing a controller where the performance of the system response can be improved.

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